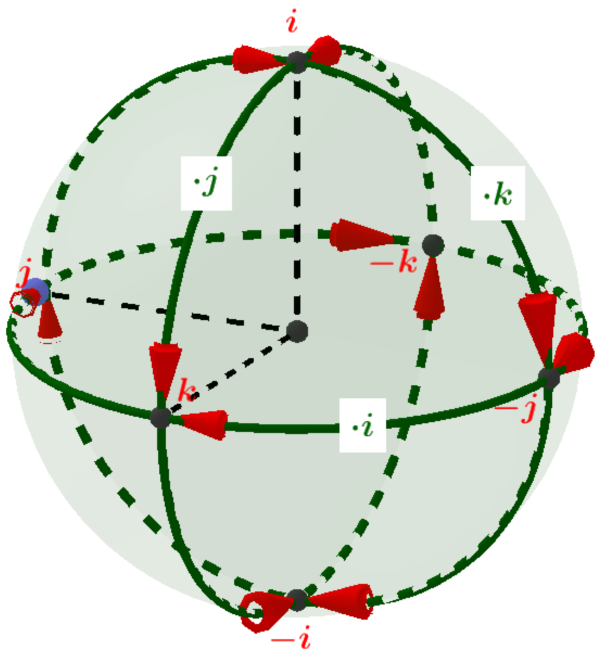


"Quaternion Ball"

A quaternion teaching-aid

The "Quaternion Ball" is a simple, costless tool for teaching school-children about the quaternion number system, particularly the equations determining how to multiply the quaternions' imaginary units i,j and k. The construction of the Quaternion Ball is based on the following geometric property of the 3D space with axes i, j, k:

In the plane given by the axes j, k, the rotation by $90°$ is given by $\cdot\, i$. Similarly, in the plane given by the axes k, i, the rotation by $90°$ is given by $\cdot\, j$.

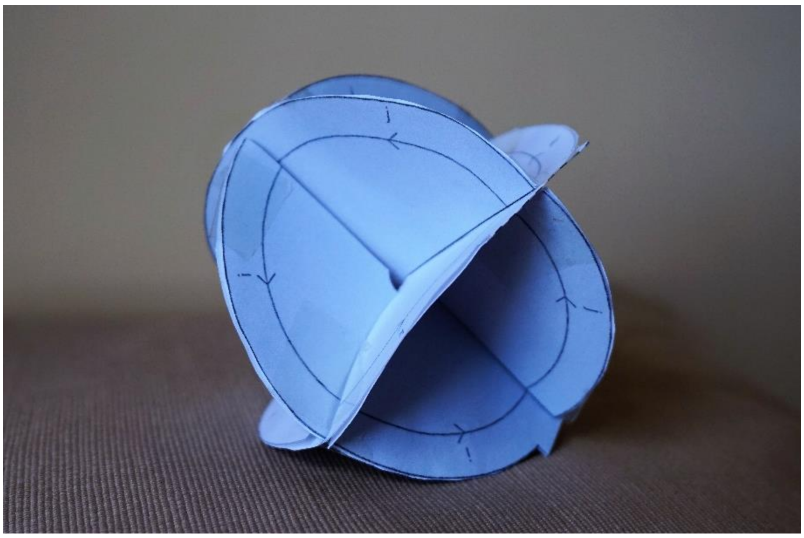
A similar statement holds for $\cdot\, k$

Note that the same is not true outside the planes described here.

Indeed, rotating a given vector $\vec{u}$ using a quaternion q involves writing $\vec{u}$ as a quaternion u and conjugating u by q: the rotated vector $= quq^{-1}$). That is, the Ball doesn't describe how quaternions are used to represent rotations in general. However, inasmuch as it uses only the 3 coordinate planes, this device can rely on rotations to give a useful and easy way of deducing the equations $i^2 = -1$, $jk = i$, etc. without using algebra.

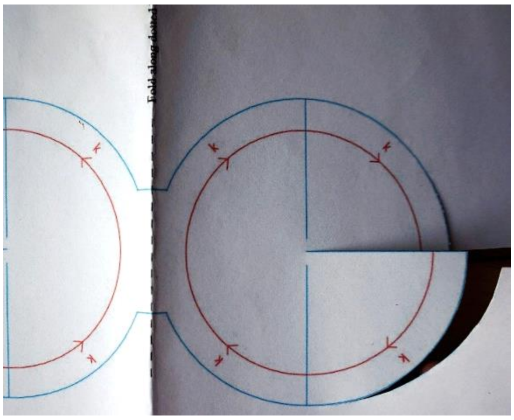
Assembly

Print and cut out the templates found at the end of this document.

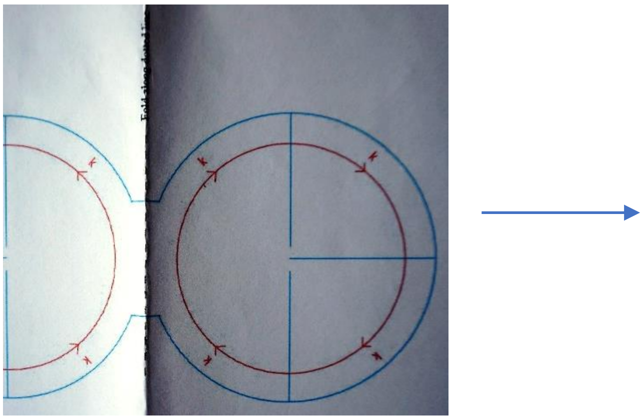


The assembled Ball

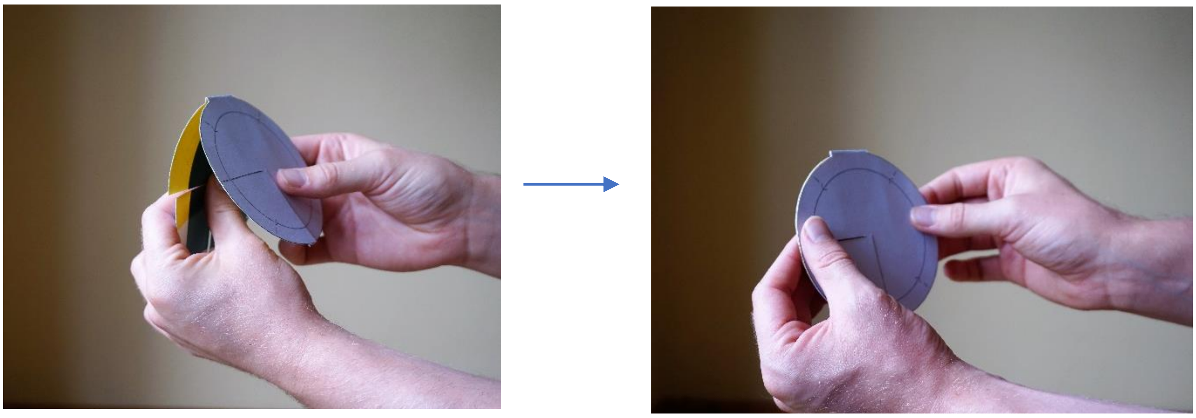
As you can see, the Ball consists of three circular cut-outs, fitted at right angles to each other. Because of this, it's best if the cut-outs are made of card or rigid paper so if your printer can print on card or rigid paper, we suggest you print the two templates on the first pages. If it can't, simply print them on normal paper, and then glue them onto two separate sheets of card. When the templates



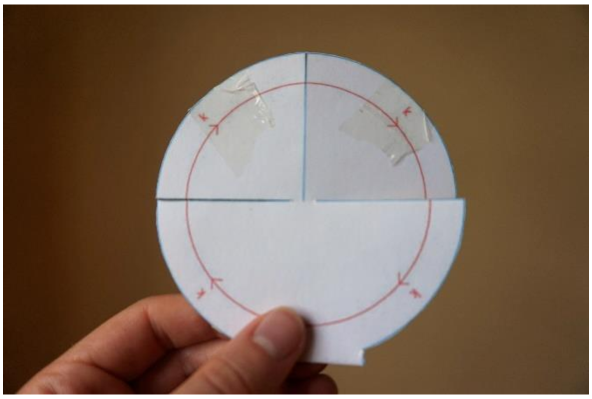
are ready, cut out the three shapes by cutting along the **blue** lines only. Where a blue line terminates on its own in the interior of one of the circles, just make a cut along the line, like this:



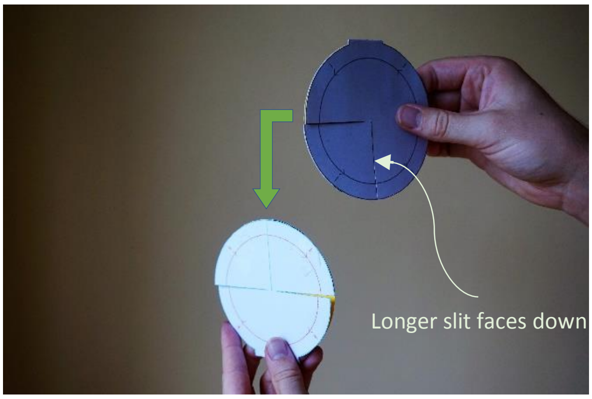
When this part is finished, you should have three independent shapes, each consisting of two joined circles. Next, fold each shape along the dotted line and uncrease, so that the two circles on any given shape overlap perfectly, with the text visible:

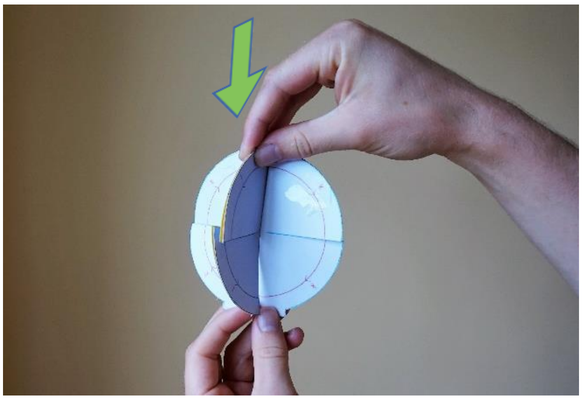


We now refer to each of the three resulting "discs" by the letter that's on it ("i-, j- and k-discs"). Hold the k-disc in front of you as shown below, such that the arrows on it point clockwise. There should be two slits in the j-disc;- a long one and a short. Hold the j-disc (as below) with the long slit pointing downwards and the short slit pointing towards you. Slide the j-disc onto the k-disc. Next, hold the i-disc horizontally, with its only slit facing away from you, so that if you look down on it, the arrows point anticlockwise. Slowly slide the i-disc into the j- and k-discs.

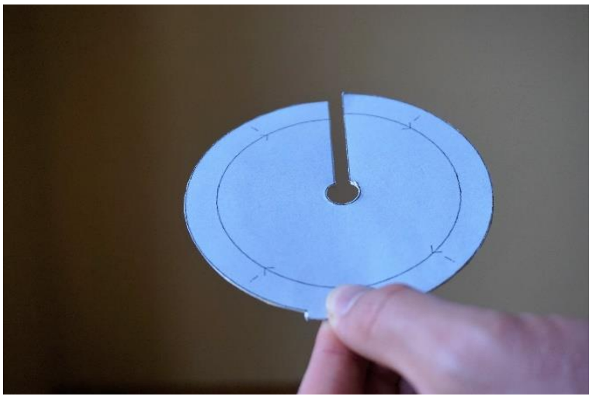


The k-disc held in front of you.

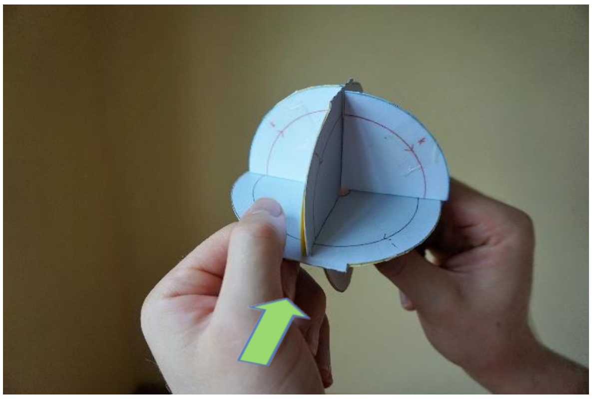




Slotting the k- and j-discs together.



The i-disc held horizontally.

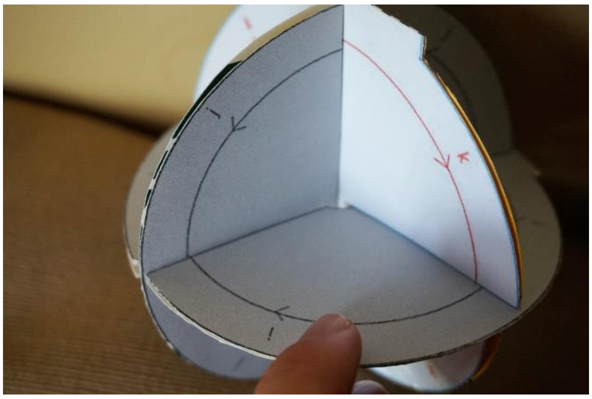


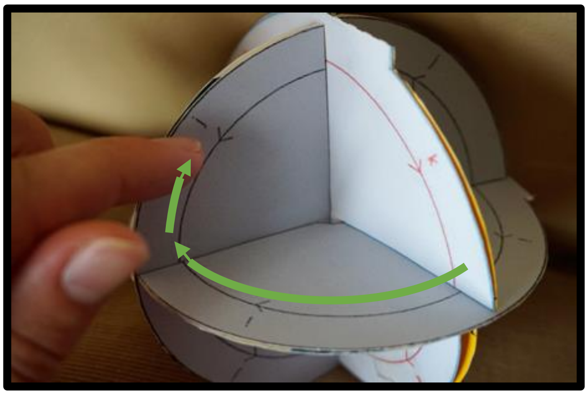
Slotting in the i-disc.

Use

The Ball helps to derive and recall the nine equations governing quaternion multiplication. When doing this, it might help to trace a fingertip along the red arcs on the Ball. Tracing a quarter circular arc in the direction of the arc's arrow corresponds to the quaternion unit printed above that arc; tracing opposite to the arrow's direction corresponds to the negative of the unit. Tracing a semi-circular arc corresponds to -1, while not moving your fingertip at all (the "identity tracing") corresponds to 1. Tracing any given quarter arc followed by another adjacent arc simply corresponds to the product of the two associated units;- below, for example, I trace an "i-arc" and then a j-arc opposite to the arrow direction, and so I've traced out the product $i(-j) = -ij$.

Taking any other route on the Ball to reach the same finish point correponds to a product that is

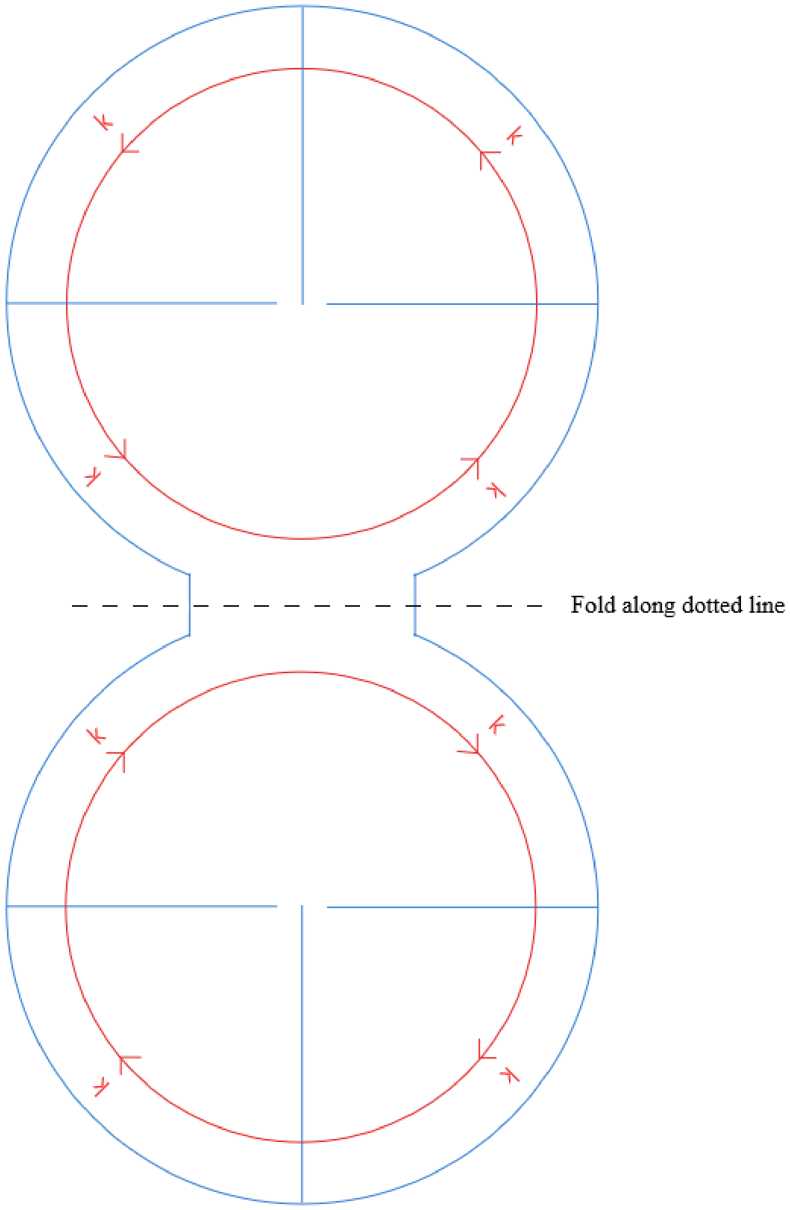


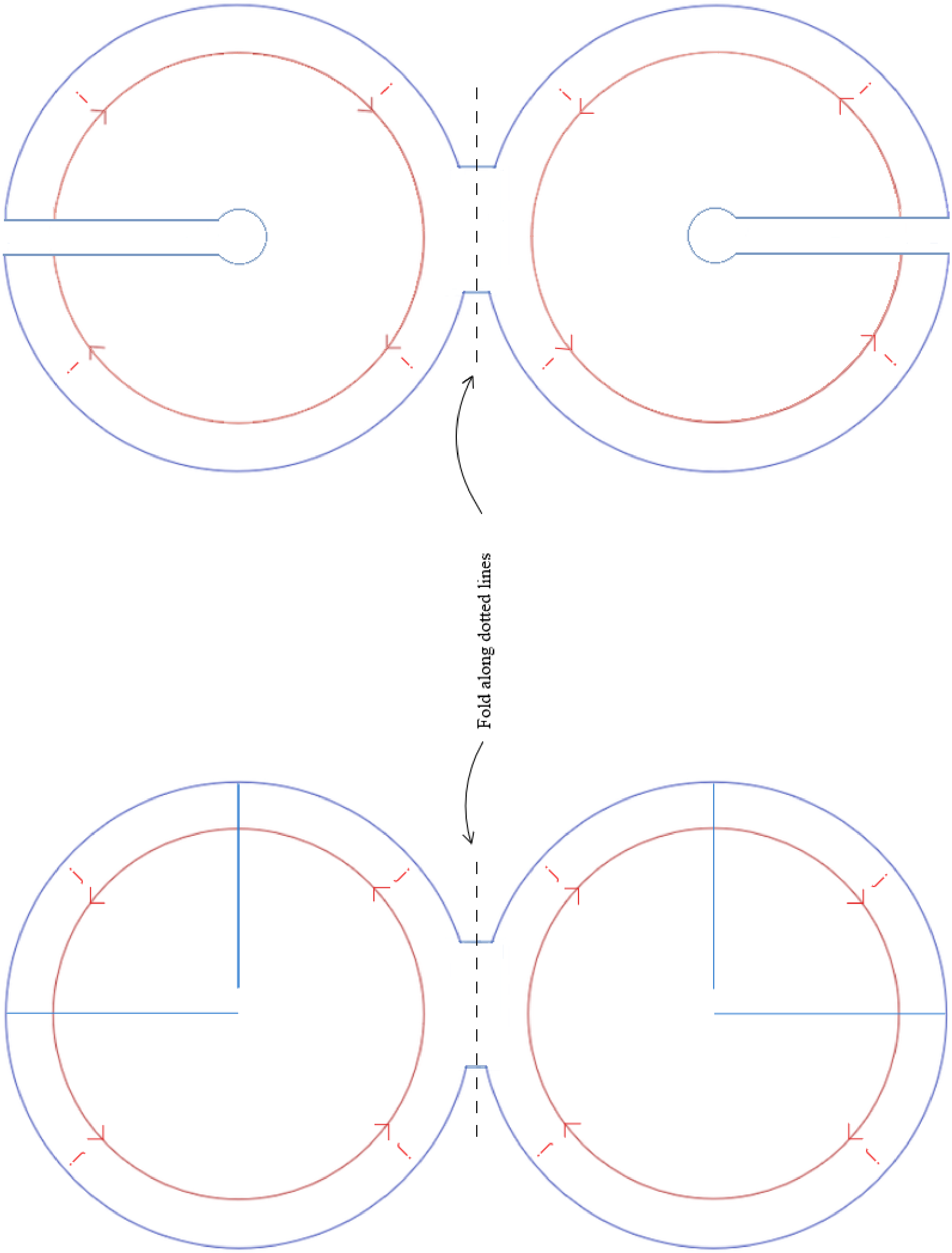


equal to $-ij$;- here, I could also have traced a k-arc opposite to the arrow, so I can say that $-ij = -k$, or $ij = k$. It can also immediately be seen that tracing out two i- , j- or k-arcs in a row is the same as a semi-circular arc, so $i^2 = j^2 = k^2 = -1$. Note that if you haven't quite assembled the Ball as explained above, you might find the Ball gives the wrong answers, like $ij = -k$. In this case, just take apart the ball and re-assemble exactly as described above.

If you feel the Ball is too flimsy, you can reinforce it by sellotaping the discs together at their edges.

Quaternion Ball Template – Cut along the blue contours and lines.




Fold along dotted lines